THERMAL MODEL-BASED DETERMINATION OF DISSOCIATION DEGREE OF HYDROGEN FLOWING IN A HOT TUBE


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A thermal model of a cylindrical thin-walled tube heated up to a high temperature by electric current has been developed. For the current-carrying tube, the heat conduction equation is solved by taking into account the tube radiation, heat exchange with the gas surrounding and flowing inside the tube, and the gas dissociation at the tube surface. The model is verified via comparison with experimental data on the hot tube temperature and electrical resistance for argon, helium, and hydrogen up to a tube temperature of 2200°C. To ground the heat exchange with the flowing gas used in the model, calculations of the gas flow in the tube have been performed using the direct simulation Monte Carlo method. The proposed thermal model allows determining the channels of distribution of the energy released at the tube as a result of Joule heating. The calculated heat balance makes it possible to estimate the degree of hydrogen dissociation at the outlet of the tube.

KEY WORDS: high-temperature tube, gas–surface heat transfer, thermal model, hydrogen dissociation

1. INTRODUCTION

Gas flows through thin heated tubes are widely used both in the field of experimental studies and in industry, in particular, for organization of molecular and atomic beams (Lukas, 2014). Thin tubes allow fulfilling uniform heating of the gas effectively using the catalytic properties of the surface due to multiple collisions with the surface to control the residence time of particles in the tube, and thus influence the gas-phase reactions. Thin tubes are used to dissociate hydrogen, oxygen, nitrogen, etc., in order to obtain active components from halogen- and carbon-containing gases (Lukas, 2014). At the present time, thin tubes are actively used in gas-phase deposition and gas-jet cleaning of surfaces. In particular, in recent years a method of synthesizing diamond structures from the gas phase has been developed, based on thermal activation of gas during the interaction with an extended activating surface when flowing through a tube (Rebrov et al., 2015, 2016; Rebrov, 2017). A distinctive feature of this approach is the use of heterogeneous processes of hydrogen dissociation in the case of multiple collisions of molecules with a hot surface (Rebrov, 2017; Rebrov and Yudin, 2016; Plotnikov and Shkarupa, 2016). This approach shows promise for increasing the degree of hydrogen dissociation at the tube outlet and more efficient delivery of the active components of the gas mixture to the substrate.

Atomic hydrogen plays an important role in many technological processes, and information about the degree of dissociation of hydrogen could contribute to their optimization. Despite the long history of the study of the hydrogen dissociation on hot metal surfaces under various conditions, the process of constructing the theory is far from complete. Therefore, experimental approaches to estimate the degree of dissociation are important. One of the approaches used to estimate the dissociation degree is the analysis of heat balance; in particular, when hydrogen flows around a wire (Langmuir and Mackay, 1914; Gat and Angus, 1993; Zheng and Gallagher, 2006; Mankelevich et al., 2014).
In the present paper, a thermal model of a cylindrical reactor heated up to 2200°C by electric current was constructed, taking into account the dissociative heat exchange with the flowing and background gases. Verification of the constructed model was carried out by comparing the calculated temperature and resistance with the experimental data. The calculated heat balance allows us to estimate the degree of dissociation of hydrogen at the outlet of the tube.

2. STATEMENT OF THE PROBLEM

In gas-jet deposition (Rebrov et al., 2016), the reactor consists of two coaxial tubes in a copper casing. Inside the tubes there are flows of the used gases (methane, hydrogen, and argon). One of the tubes is heated by electric current to high temperatures, which ensures the effective dissociation of hydrogen on the tungsten surface of the tube. In this paper, we consider a simplified model of a reactor consisting of one current-carrying tube.

A tube with a diameter of 6 mm and a length of 155 mm is considered as the model reactor. A schematic representation of the experimental reactor is shown in Fig. 1. The tube consists of two parts: a molybdenum tube (110 mm long) and tungsten foil tube (38 mm long). A tube of tungsten foil is obtained by rolling a plate with a length of 55 mm and width of 21 mm, such that part of the foil tube with a length of 10 mm overlaps the molybdenum tube. To ensure good electrical contact between the wires and the tungsten foil, a molybdenum cylinder with a length of 7 mm is inserted into the foil tube from the open edge. The entire tube is mounted vertically in a chamber with a diameter of 340 mm. The thickness of the tungsten foil is 31 μm, and the wall thickness of the molybdenum tube is 0.5 mm. The tube is supplied with electric current at current strength up to 130 A, which heats the tungsten foil to high temperatures (up to 2200°C). The electric current flows to the tungsten foil tube from one side through the molybdenum tube and from the other side through two molybdenum wires with a diameter of 1.2 mm and length of 65 mm. In the experiment, the resistance of the central part of the foil tube with a length of 20 mm is measured (in which tantalum rings for measuring voltage are fixed at a distance of 15 mm and 35 mm from the open edge of the tube). Also, the pyrometer measures the temperature of the tube at different points.

The main experiments were performed for hydrogen. To develop and verify the model, experiments with helium and argon were also performed. Two modes were used:

1. The tube is in a chamber with gas at rest; and
2. Gas is supplied through the tube at a flow rate of 1500 standard cubic centimeters per minute (sccm).

The pressure in the chamber in all of the experiments was maintained at a level of 20 Torr.

3. MATHEMATICAL MODEL

For the current-carrying tube, the non-stationary heat conduction equation is solved by taking into account the tube radiation, heat exchange with the gas surrounding and flowing inside the tube, and its dissociation.
\[ c g S \frac{\partial T}{\partial t} = \frac{I^2 \rho}{S} + \lambda S \frac{\partial^2 T}{\partial x^2} - 2\pi R\epsilon_\sigma (T^4 - T_\infty^4) - \alpha_{out} \cdot 2\pi R\epsilon_\sigma (T^4 - T_\infty^4) - H_{gas}(T) - H_{emiss}(T) - H_{flow}(T) - H_{dis}(T) \]  

(1)

where \( c \) is specific heat of the metal; \( g \) is the metal density; \( T(x, t) \) is the tube temperature; \( S = \pi \Delta(2R + \Delta) \) is the tube cross section (0.57 mm² for tungsten and 10.21 mm² for molybdenum); \( \Delta \) is the tube wall thickness (31 \text{ µm for tungsten and 0.5 mm for molybdenum}); \( R = 3 \text{ mm} \) is the tube radius; \( I \) is current strength; \( \rho \) is the resistivity; \( \lambda \) is the coefficient of the thermal conductivity of the metal; \( \epsilon \) is the tube emissivity; \( \epsilon_{Cu} = 0.4 \) is the copper emissivity; \( R_\infty = 170 \text{ mm} \) is the radius of the chamber; \( \sigma \) is the Stefan–Boltzmann constant; \( T_\infty = 27^\circ C \) is the temperature of the chamber surface; \( \alpha_{out} \) is the probability of the emitted photon escaping through the end opening (see the left-hand side in Fig. 1) when it is emitted from the inner surface of the tube [see Eq. (A3) in Appendix A]; \( H_{gas}(T) \) is the heat flux from the tube to the surrounding space [see Eq. (3)]; \( H_{emiss}(T) \) is the heat flux caused by the radiation between the tube sections with different temperatures [see Eq. (4)]; \( H_{flow}(T) \) is the heat flux caused by heating the gas flowing in the tube [see Eqs. (5) and (6)]; and \( H_{dis}(T) \) is the heat flux caused by the dissociation of gas [see Eq. (8)]. The properties of the substances used in the calculations are presented in Appendix B.

4. HEAT EXCHANGE WITH THE SURROUNDING GAS

Consideration of the heat exchange between the tube and the surrounding gas (taking into account the free convection), is based on the use of the Nusselt criterion \( Nu_D \). The heat flux to the surrounding space is determined by the expression

\[ H_{gas}(T) = a \cdot 2\pi R (T - T_\infty) \]  

(3)

where \( a = h \cdot Nu_D / D \) is the heat transfer coefficient; \( h \) is the coefficient of thermal conductivity of the gas; and \( Nu_D \) is the Nusselt number determined by the diameter of the tube, \( D = 2R \).

The Nusselt number is uniquely determined by the Rayleigh number \( Ra_D = Gr_D \cdot Pr \) (Martynenko and Khramtsov, 2005) and the ratio \( D/L_t \), where \( L_t \) is the length of the tube. Here, \( Gr_D = g_{grav} \beta D^3 \Delta T/v^2 \) is the Grashof number; \( g_{grav} \) is the free fall acceleration; \( \beta \) is the temperature coefficient of volumetric expansion; \( v = \eta / \gamma g \) is the kinematic viscosity coefficient; \( \eta \) is the dynamic viscosity coefficient; \( Pr = v/\alpha_t \) is the Prandl number; and \( \alpha_t = \lambda (g C_p) \) is the thermal diffusivity. The resulting dependence of the Rayleigh number on the tube temperature is shown in Fig. 2. The coefficients of viscosity and thermal diffusivity were determined under the average temperature \( T_{av} = (T + T_\infty)/2 \). For argon, we have \( Ra_{D} \sim 10^{-2} \), and for hydrogen and helium, we have \( Ra_{D} \sim 10^{-5} \).

Reliable data on the free convective heat transfer for the considered conditions (large temperature difference \( \Delta T \) and low gas pressure) are missing. Known empirical dependencies give values of the Nusselt number from 0.3 to 1 (see Fig. 3 and Table 1). It is possible to estimate the limiting values of the Nusselt number in a gas at rest without convection. For a sphere in an unbounded space, \( Nu = 2 \) (Rebrov, 1966). The Nusselt number for two coaxial cylinders with diameters \( D_2 > D_1 \) of unlimited length is equal to \( Nu_D = 2/\ln (D_1/D_2) \) (Rebrov, 1966), which for our conditions (\( D_1 = 6 \text{ mm} \) and \( D_2 = 340 \text{ mm} \)) gives the value \( Nu_D = 0.49 \). The Nusselt number without taking into account convection is between the dependences of the heat exchange of a sphere and a circular cylinder of unlimited length, which for our conditions gives a range of \( 0.49 < Nu_D < 2 \). The analysis of our experimental data for quiescent argon, helium, and hydrogen showed that good agreement between the calculation and experiments occurs at \( Nu_D = 0.8 \), which agrees fairly well with the literature data presented in Fig. 3 and Table 1. It is this value that was used further in all of our calculations.

Since the tube is heated nonuniformly, it is important to take into account the radiative heat exchange between different parts of the tube. For this, there is the integral term in Eq. (1)
FIG. 2: The Rayleigh number as a function of temperature for different gases

FIG. 3: The Nusselt number as a function of the Rayleigh number based on various empirical dependencies

TABLE 1: Values of Nusselt number \( \text{Nu}_D \) estimated using various empirical dependences

<table>
<thead>
<tr>
<th>Formula (Reference)</th>
<th>( \text{Nu}_D ) Ar: ( \text{Ra}_D = 10^{-2} ), ( \text{Ra}_D \text{D}/L_t = 10^{-3} )</th>
<th>( \text{He}, \text{H}_2: \text{Ra}_D = 10^{-5} ), ( \text{Ra}_D \text{D}/L_t = 10^{-6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Nu}_D = \text{Ra}_D^{0.11} ) ( 10^{-6} &lt; \text{Ra}_D &lt; 10^{-2} ) (Mueller, 1942; Morgan, 1975)</td>
<td>0.603</td>
<td>0.282</td>
</tr>
<tr>
<td>( \text{Nu}_D = 2\left(\ln\left{\frac{4.47}{\text{Ra}_D \text{D}/L_t}\right}\right)^{0.26} ) ( 10^{-11} &lt; \text{Ra}_D \text{D}/L_t &lt; 10^{-4.5} ) (Kyte et al., 1953)</td>
<td>0.601</td>
<td>0.392</td>
</tr>
<tr>
<td>( \text{Nu}_D = 0.93 \left(\text{Ra}_D \text{D}/L\right)^{0.05} ) ( \text{Ra}_D \text{D}/L_t &lt; 0.05 ) (Nagendra et al., 1970)</td>
<td>0.658</td>
<td>0.466</td>
</tr>
<tr>
<td>( \text{Nu}_D = 1.19 \text{Ra}_D^{0.025} ) ( 10^{-5} &lt; \text{Ra}_D &lt; 1 ) (Rebrov, 1966)#</td>
<td>1.025</td>
<td>0.819</td>
</tr>
<tr>
<td>( \text{Nu}_D = \left[0.98 - 0.01 \left(\log \text{Ra}_D\right)^2\right] \text{Ra}_D^{0.14 + 0.015 \log \text{Ra}_D} ) ( 10^{-7} &lt; \text{Ra}_D &lt; 4 \times 10^8 ) (Rebrov, 1961)*</td>
<td>0.566</td>
<td>0.345</td>
</tr>
<tr>
<td>( \text{Nu}_D = 0.675 \text{Ra}_D^{0.08} ) ( 10^{-10} &lt; \text{Ra}_D &lt; 10^{-2} ) (Morgan, 1975)*</td>
<td>0.517</td>
<td>0.346</td>
</tr>
</tbody>
</table>

\#The dependence is based on the presented experimental data.
*Data for the horizontal cylinder.

\[
H_{\text{emis}}(T) = \int_0^{x_{\text{max}}} \alpha_{\text{emis}} \left(\frac{x'}{R}\right) \cdot \sigma \left\{\varepsilon \left[T(x')\right] T(x')^4 - \varepsilon \left[T(x)\right] T(x)^4\right\} 2\pi R dx'
\]  \hspace{1cm} (4)

where \( \alpha_{\text{emis}} \) is the fraction of photons absorbed on the inner surface of a cylinder of the unit length at a distance \( x' \) from emission point \( x \) [see Eq. (A4)].

Figures 4 and 5 present the calculation results and experimental data for argon at rest. Figure 4 shows the tube temperature profiles for currents of 80 and 100 A compared with pyrometric measurements. Taking into account the
radiative heat transfer inside the tube by using formula \( \text{(4)} \) leads to a slight decrease in temperature. Figure 5 shows the corresponding dependences of the tube resistance on the current strength. One can see good agreement between the calculations and the experimental measurements.

5. HEATING GAS FLOWING THROUGH THE TUBE

To take into account the heating of the gas flowing through the tube, we introduce the gas temperature \( T_{\text{gas}} \). Since the direction of the gas flow is negative in the \( x \) coordinate system, it is convenient to introduce the \( y \) coordinate as \( y = L_{\text{tube}} - x \), where \( L_{\text{tube}} = 155 \text{ mm} \) is the tube length. We assume that at the inlet to the tube \((y = 0)\) the temperature of the gas is \( T_{\text{gas},0} = 27^\circ \text{C} \). By analogy with the relaxation of gases, an increase in the gas temperature along the tube will be determined by the equation

\[
\frac{dT_{\text{gas}}}{dy} = \frac{T - T_{\text{gas}}}{L} \quad \text{(5)}
\]

Here, \( L \) is the length of the relaxation of the gas temperature to the wall temperature, which is defined as the distance over which the temperature difference decreases by the factor \( e \). For a constant tube temperature \( T(y, t) = T_{\text{tube}} \), solving this equation gives the temperature distribution of the flowing gas along the tube

\[
T_{\text{gas}} = T_{\text{tube}} + (T_{\text{gas},0} - T_{\text{tube}}) \exp \left(-\frac{y}{L}\right)
\]

Knowing the temperature of the gas, it is possible to determine the heat flux aimed at heating the gas flowing in the tube as

\[
H_{\text{flow}} = C_{\text{gas}} G \frac{dT_{\text{gas}}}{dy} \quad \text{(6)}
\]

where \( C_{\text{gas}} \) is the gas heat capacity; and \( G \) is the gas flow rate.

To verify the proposed approach for consideration of the heating of the gas, we performed calculations on the gas flow in the tube using the direct simulation Monte Carlo (DSMC) method (Bird, 1994). Presently, the DSMC method
is the numerical tool most used in solving problems of rarefied gas dynamics. It consists of modeling the movement of individual particles by taking into account their interaction with each other and the surrounding surfaces. More detailed descriptions of the algorithm used are given in Morozov et al. (2016) and Rebrov et al. (2018). To describe the interaction of particles with the surface, a model of specular-diffuse reflection was used, which is determined by the energy accommodation coefficient $c$ (Bird, 1994). The gas flow was simulated through a cylindrical tube in a chamber filled with gas at a pressure of 20 Torr. The temperature profile of the tube was set from the preliminary calculation using the thermal model. The length of the relaxation temperature ($L$) depends on the accommodation coefficient $c$. Figure 6 presents as an example the results of the calculations for argon flow in the tube. Based on the DSMC calculations, the values of $L$ were selected that corresponded to the accommodation coefficients. It can be seen that a decrease in the accommodation coefficient leads to an increase in $L$. Fairly good agreement was obtained between the DSMC calculation and the solution to Eq. (6) [Fig. 6(a)]. In this case, the accommodation coefficient (and accordingly distance $L$) rather weakly affects the temperature profile [Fig. 6(b)].

Figure 7 presents similar data for hydrogen. Based on the experimental data, the length of the gas temperature relaxation was selected, which turned out to be close to a similar value for argon: $L = 20$ mm. As shown by the DSMC calculation [Fig. 7(a)], this distance corresponds to a much smaller coefficient of accommodation of gas on the surface compared with argon: $\alpha_c = 0.02$. Indeed, the mass of the hydrogen molecule is 20 times less than the mass of the argon atom; therefore, for hydrogen, the thermal velocity of the particles, and accordingly the frequency of collisions with the surface, will be $\sqrt{20} \approx 4.5$ times higher than for argon. To preserve the energy flux from the wall, the accommodation coefficient should be 4.5 times less than for argon. The resulting value of the accommodation coefficient is in good agreement with the experimental data on accommodation on clean surfaces (Wachman, 1966; Goodman and Wachman, 1967).

6. GAS DISSOCIATION AT THE TUBE SURFACE

Hydrogen dissociation due to heterogeneous reactions on a tungsten surface is observed at surface temperatures above 1600 K (Smith and Fite, 1962; Koschmieder and Raible, 1975; Otsuka et al., 1995; Zheng and Gallagher, 2006). From analysis of the energy balance, one can determine the total number of atoms ($N_{H_2}$) and molecules ($N_{H_2}$) of hydrogen at the outlet of a hot tube. These data allow calculating the effective degree of dissociation of hydrogen molecules as

![Figure 6](image-url)
Thermal Model-Based Determination of Dissociation Degree

FIG. 7: Energy flux from the tube to the flowing gas (a) and the tube temperature profile (b) for hydrogen at a current of 100 A: DSMC calculations for accommodation coefficients $\alpha_c = 0.02$ and model Eq. (6) for the distance $L = 20$ mm; the results of pyrometric measurements of the temperature profile are also presented.

$\alpha = N_H/(N_H + 2N_{H_2})$. It should be noted that based on this approach it is not possible to analyze separately the reactions of dissociation and recombination.

The number of collisions of hydrogen molecules with a fragment of the tube wall with an area $S$ per unit of time can be estimated as

$$N_{col} = \frac{1}{4} n_{\text{tube}} u_T S = \frac{p_{\text{tube}} S}{\sqrt{2 k T m}} \quad (7)$$

where $n_{\text{tube}}$ and $p_{\text{tube}}$ are the density and pressure inside the tube, respectively; and $u_T$ is the average thermal velocity of the molecules. The DSMC calculations show that the pressure inside the hot part of the tube with the subsonic flow for the conditions under consideration (background pressure 20 Torr; flow rate 1500 sccm) varies slightly along the tube and exceeds the background pressure by about 2 Torr, i.e., $p_{\text{tube}} \approx 22$ Torr. It is assumed that the number of hydrogen dissociation acts inside and outside the tube is the same. In this case, the heat flux in Eq. (1), due to gas dissociation, is determined by the expression

$$H_{\text{dis}} = E_{\text{dis}} \cdot \frac{2 \pi R_{\text{tube}} p_{\text{tube}} + p_{\infty}}{2 k T m} h_{\text{dis}}(T) \quad (8)$$

where $h_{\text{dis}}(T)$ is the probability of effective dissociation when a molecule collides with the surface. In the previous studies by Redman et al. (1999), Umemoto et al. (2002), Zheng and Gallagher (2006), and Comerford et al. (2009), it was shown that when activated by a hot wire, the rate of formation of hydrogen atoms near the surface is determined by an energy of 2.5 eV, which is much less than the dissociation energy of hydrogen, $E_{\text{dis}} = 4.5$ eV. This difference was explained within the framework of the two-stage gas-phase surface mechanism of catalytic dissociation (Comerford et al., 2009). At the same time, it was shown by Comerford et al. (2009) that, in fact, the activation energy on the catalytic surface equals $E_{\text{cat}} = 2$ eV. Based on the results of Comerford et al. (2009), we will further assume that $h_{\text{dis}}(T)$ is given as a function of

$$h_{\text{dis}} = A_{\text{dis}} \exp \left( -E_{\text{cat}}/kT \right) \quad (9)$$

where $A_{\text{dis}} = 20$ is the constant that was chosen in order to best describe experimental dependence $R = R(I)$.

Figure 8 shows the dependences of the resistance and the temperature of the tube and the degree of dissociation $\alpha$ of the flowing hydrogen on the current strength. Note that the calculation without taking into account the dissociation does not allow correctly describing the experimental data for large values of the current strength; taking into account
the heating of the flowing gas and dissociation leads to a significant decrease in the tube temperature, and accordingly the resistance. One can see good agreement between the calculation results and the experimental measurements [Fig. 8(b)]. At a current of 130 A, the degree of dissociation at the tube outlet reaches a value of $\alpha = 11.3\%$. The DSMC calculations for gas flow in the tube for a temperature profile corresponding to 130 A, taking into account the effective dissociation probability by formula (9), give a close value of the degree of dissociation at the tube outlet, $\alpha = 13\%$.

Figure 9 shows which channels distribute the energy released on the tube as a result of Joule heating. It can be seen that for large values of the current strength up to 55% of the energy is released via radiation.

The probability of the effective dissociation $h_{\text{dis}}(T)$ calculated by formula (9) can be compared with the data on hydrogen dissociation on hot filaments (Comerford et al., 2009; Mankelevich, 2013). Using an experimental/analytical
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approach, Mankelevich (2013) derived detailed data on the surface dissociation rate. Knowing these data, one can calculate the flux of hydrogen molecules on the wall and estimate the effective probability of dissociation. For example, let us estimate the probability for the filament temperature $2127^\circ C$. It is assumed that the gas temperature is lower than the filament temperature by 490 K, i.e., $T_{gas} = 1637^\circ C$. For pressure of 20 Torr, the gas density can be calculated as $n = p/(kT_{gas}) = 10^{23} m^{-3}$. The flux of hydrogen molecules colliding with the surface is $\Psi = (1/4) n \sqrt{8kT_{gas}/(\pi m_{H_2})} = 1.14 \cdot 10^{26} s^{-1} m^{-2}$. Mankelevich (2013) estimated the catalytic H atom production as $Q = 6.88 \cdot 10^{23} s^{-1} m^{-2}$. Hence, the probability of dissociation can be estimated as $h = Q/(2\Psi) = 0.003$ (here, coefficient 2 is due to the fact that one molecule gives two hydrogen atoms during dissociation). Using formula (9), this value is consistent in the order of magnitude with a probability of 0.0013.

7. CONCLUSIONS

A one-dimensional model of the temperature distribution along a hot tube has been developed. The model takes into account the tube radiation, heat exchange with the gas surrounding and flowing inside the tube, and the gas dissociation at the tube surface. The results of the numerical calculations are in good agreement with the results of the experimental measurements of the tube temperature and resistance. The developed model allows calculating the temperature of the tube surface for a given current strength and gas flow rate. The calculated heat balance allows estimating the degree of dissociation of hydrogen at the outlet of the tube.

ACKNOWLEDGMENTS

The theoretical part of the study was carried out under a state contract with the Institute of Thermophysics, Siberian Branch of the Russian Academy of Sciences (AAAA-A17-117030110017-0); the experimental part of the study was financially supported by the Russian Foundation for Basic Research (Project Nos. 18-29-19069 and 19-08-00533).

REFERENCES


APPENDIX A. RADIATION HEAT TRANSFER CONFIGURATION FACTORS

In the literature, analytical solutions for the configuration factors for radiation heat transfer are known. The fraction of radiation emitted through the end walls of a cylinder of length \( L \) and radius \( R \) is (Leuenberger and Person, 1956)

\[
F = \frac{1}{2} \left( \sqrt{4 + \frac{L^2}{R^2}} - \frac{L}{R} \right) = \frac{1}{2} \left( \sqrt{4 + x^2} - x \right)
\] (A.1)
where \( x = L/R \). Let us determine the probability that a photon emitted at a distance \( x \) from the end wall will escape from the cylinder. Let \( \alpha_{\text{out}}(x) \) be the escape probability, and then the fraction of the integral radiation emitted through the ends will be

\[
F(L) = \frac{1}{L} \int_0^L \alpha_{\text{out}}(x) \, dx \tag{A.2}
\]

From here, we get the escape probability through one end as

\[
\alpha_{\text{out}}(x) = \frac{1}{2} \frac{d[xF(x)]}{dx} = \frac{1}{2} \left( \frac{2 + x^2}{\sqrt{4 + x^2}} - x \right) \tag{A.3}
\]

Thus, we can derive the probability of a photon to be absorbed on the inner surface of the cylinder at a distance \( x \) from the point of the photon emission as

\[
\alpha_{\text{emis}}(x) = -\frac{d\alpha_{\text{out}}(x)}{dx} = \frac{1}{2} \left( -x \left( 6 + x^2 \right) \right) (4 + x^2)^{3/2} \tag{A.4}
\]

**APPENDIX B. THERMO PHYSICAL PROPERTIES OF SUBSTANCES**

Table B1 presents the thermophysical properties of tungsten and molybdenum, obtained by approximation of the experimental data from Marmer et al. (1967). In the formulas, temperature \( T \) is given in degrees Celsius. Table B2 presents the thermophysical properties of the gases used. The expressions for viscosity are obtained by approximation of the data from Grigoriev and Meilikhov (1991). In the formulas, temperature \( T \) is given in degrees Kelvin. The heat capacity \( (C_{\text{gas}}) \) was taken equal to \( 2.5R \) for argon and helium and \( 3R \) for hydrogen (where \( R \) is the universal gas constant).

**TABLE B1:** Thermophysical properties of tungsten and molybdenum

<table>
<thead>
<tr>
<th>Property</th>
<th>Tungsten</th>
<th>Molybdenum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical resistivity ( \rho ) ( (10^{-8} , \text{Om} \cdot \text{m}) )</td>
<td>( 4 \cdot (1 + 0.0069T + 4.63 \cdot 10^{-7}T^2) )</td>
<td>( 3.5(1 + 0.0086T) )</td>
</tr>
<tr>
<td>Coefficient of heat conductivity ( \lambda ) ( [\text{W/}(\text{m} \cdot \text{K})] )</td>
<td>( 156 - 0.062T + 2.02 \cdot 10^{-5}T^2 ) (- 2.3 \cdot 10^{-9}T^3 )</td>
<td>( 151 - 0.062T + 1.15 \cdot 10^{-5}T^2 ) (- 1.15 \cdot 10^{-9}T^3 )</td>
</tr>
<tr>
<td>Emissivity ( \varepsilon )</td>
<td>0.0013 + 0.000194T - 2.4 \cdot 10^{-8}T^2</td>
<td></td>
</tr>
<tr>
<td>Density ( \rho ) ( (\text{g/cm}^3) )</td>
<td>19.35</td>
<td>10.22</td>
</tr>
</tbody>
</table>

**TABLE B2:** Thermophysical properties of argon, helium, and hydrogen

<table>
<thead>
<tr>
<th>Property (Coefficient)</th>
<th>Argon</th>
<th>Helium</th>
<th>Hydrogen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic viscosity ( \eta ) ( (10^{-6} , \text{Pa} \cdot \text{s}) )</td>
<td>( 10 + 0.051T ) (- 7.76 \cdot 10^{-6}T^2 )</td>
<td>( 8.9 + 0.041T ) (- 5.27 \cdot 10^{-6}T^2 )</td>
<td>( 3.4 + 0.02T ) (- 2.28 \cdot 10^{-6}T^2 )</td>
</tr>
<tr>
<td>Heat conductivity ( \lambda ) ( [\text{W/}(\text{m} \cdot \text{K})] )</td>
<td>( 0.005465 + 4.729 \cdot 10^{-5}T ) (- 1.111 \cdot 10^{-8}T^2 + 1.599 \cdot 10^{-12}T^3 ) (Chen and Saxena, 1975)</td>
<td>( 0.0476 + 3.62 \cdot 10^{-4}T ) (- 6.18 \cdot 10^{-8}T^2 + 7.18 \cdot 10^{-12}T^3 ) (Vargaftik and Yakush, 1977)</td>
<td>( 0.05388 + 3.815 \cdot 10^{-4}T ) (Vargaftik and Vasilevskaia, 1975)</td>
</tr>
<tr>
<td>Volumetric expansion ( \beta ) ( (10^{-3} , \text{K}^{-1}) )</td>
<td>3.676</td>
<td>3.658</td>
<td>3.659</td>
</tr>
</tbody>
</table>