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EFFECTIVE PARAMETRIZATION FOR RELIABLE RESERVOIR PERFORMANCE PREDICTIONS

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The purpose of reservoir modeling and simulation is to predict reservoir performance for development and depletion planning. Despite decades of research, efficient and reliable reservoir performance predictions are still a challenge in practice. In this paper, we present an overview of reservoir modeling as it is commonly practiced today and the challenges it faces. More specifically, we focus on the challenges posed by the large amount of uncertainty inherent in the characterization of reservoirs that are heterogeneous at multiple scales. We discuss the practical implications of these challenges and recent developments toward addressing them. In particular, we examine the need for effective parametrization of geologic concepts and related recent advances in parametrization and parameter reduction techniques, including their advantages and limitations. Using numerical examples from two different depositional environments, we show that effective parametrization can be achieved by taking advantage of the geologic hierarchy underlying most geologic concepts and a general understanding of the impact of geologic features on fluid flow. Finally, we propose an approach to systematically derive fit-for-purpose parametrization for practical reservoir modeling problems.

KEY WORDS: *heterogeneous random media, uncertainty quantification, multiscale modeling, porous media flow, reservoir engineering, multiscale estimation*

1. INTRODUCTION

When narrowly defined, reservoir modeling refers to the construction of three-dimensional digital models that represent a given hydrocarbon reservoir. The simulation of fluid flow on such models is referred to as reservoir simulation. In the following, we adopt the broader view of reservoir modeling that includes the construction, evaluation, and calibration of reservoir models.

Reservoir modeling are commonly used to predict hydrocarbon production from a petroleum reservoir over time, which is also referred to as reservoir performance. The predictions are used for development and depletion planning; they are often the basis for large investment decisions. Thus, efficient and reliable reservoir performance predictions are critical to the upstream business in the petroleum industry.

Reservoir modeling is subject to a tremendous amount of uncertainty due to limited data available. Seismic imaging is commonly used to determine large-scale structures of a reservoir, such as major hydrocarbon intervals and faults. Seismic resolution, however, is often insufficient to resolve geologic features such as thin shale barriers, subseismic faults, and fractures that may strongly impact fluid flow in the reservoir. Uncertainty is also inherent in the seismic inversion process because a seismic signal is band limited and needs to be supplemented with additional information that is based on geologic interpretations. For example, the conversion of seismic data from the time domain (where the seismic signal is recorded) to the depth domain (where the reservoir structure physically resides) requires calibrated rock physics models that depend on geologic interpretations of the distribution of different reservoir rock types. Moreover, seismic data do not provide direct measurements of reservoir porosity and permeability, the key controlling factors on reservoir performance. In contrast, well logs and core analysis provide detailed information of rock and fluid properties along the well tracks, but they both lack reservoir coverage and are carried out sparsely in

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the field. Therefore, it is usually necessary for a reservoir model to incorporate interpreted or conceived geologic descriptions in order to extrapolate measurements at the wells throughout the reservoir. The descriptions usually contain lots of uncertainty. We shall call these descriptions geologic concepts (see Section 2 for further explanation).

Given the uncertainty, a reservoir performance prediction should be accompanied by robust uncertainty and risk assessments. In other words, the prediction should not be a single prediction but a realistic range of predictions. Here, by “realistic” we mean that the range of predictions are based on a set of reservoir models (not a single model) that are consistent with the underlying geologic concepts and are calibrated against measured static (e.g., well logs) and dynamic (e.g., production at wells) field data. At the minimum, the range should cover the actual production profile. Moreover, the geologic scenarios corresponding to the high- and low-side predictions should be identified. These scenarios provide the basis for planning reservoir surveillance and risk mitigation. In the following, we will loosely call a prediction reliable if it satisfies those criteria.

Reliably and efficiently predicting reservoir performance has been a long-standing challenge in the petroleum industry. One of the main difficulties is the large amount of uncertainty, coupled with the fact that reservoirs are often strongly heterogeneous at multiple scales. Despite decades of research, efficient and reliable reservoir performance predictions are still a challenge in practice. The purpose of this paper is to review the challenges posed by the uncertainty, heterogeneity, and recent technical advances toward addressing them. We are particularly interested in effective parametrization of reservoir models, which is at the very center of the challenge.

This paper is organized as follows. In Section 2, we present an overview of the reservoir modeling practice currently adopted by the industry and analyze the challenges it faces. This is followed by a closer look at how reservoir models, especially geologic concepts, are parameterized and various parameter reduction techniques recently developed. We point out the advantages and limitations of these techniques. Then in Section 4 we propose a parametrization method based on geologic hierarchy and local multiscale representation. Both ideas are demonstrated by using numerical examples. Finally, some concluding remarks are made in Section 5.

2. RESERVOIR MODELING OVERVIEW

Reservoir modeling is a complicated multidisciplinary endeavor. In the following, an overview of the main steps of reservoir modeling is provided. It is not meant to be a review of reservoir modeling literature or detailed procedures and methodologies. Instead, we emphasize on how information and uncertainty propagate through the process. Along with the steps, we explain the practical challenges faced by reservoir modeling. Before the overview, we first give a brief introduction to geologic concepts and their role in reservoir modeling.

2.1 Geologic Concepts

A geologic concept describes the general attributes of a class of geologic scenarios for a certain depositional environment (e.g., fluvial channels, deep-water fans, carbonate platforms, etc). Mathematically, a geologic concept should be understood as a parameterized conceptual model of a group of related geologic features in a depositional environment. A concept typically consists of several environments of deposition (EODs) (see, e.g., [1]), which are related spatially. Each EOD contains a mixture of different types of sedimentary rocks or facies, which are often classified based on core observations and measurements. In clastic environments, the grain size and sorting are the main factors determining the type of a rock. The porosity and permeability within each facies typically are well correlated. For carbonates, the facies characterization is much more complicated because carbonate deposits often form from many different biological and chemical processes, and the facies are less of a control on porosity and permeability. For both clastic and carbonate rocks, especially carbonates, diagenesis may significantly change the porosity and permeability. Thus, facies and diagenesis together characterize reservoir rock types (RRTs), which are used to control the modeling of porosity, permeability, and other rock and fluid properties. (For the purpose of this paper, we will use facies and rock types synonymously.) The spatial relationships and distribution patterns of the EODs and facies are a crucial part of a geologic concept and also challenging to parameterize.

Geologic concepts are usually based on years of study of outcrops and play a central role in reservoir geology. Recently, numerical process-based modeling [2] and tank experiments (see, e.g., [3]) have also been used to better understand sedimentary processes and, hence, the spatial relationships and patterns of sediment distributions.

In reservoir modeling, geologic concepts are used to define reservoir models at subseismic scales. They are used to interpolate/extrapolate sparse well data throughout the reservoir. Without seismically derived rock type distributions, the geologic concepts are the only control on the spatial distribution of different rock-types. More importantly, subseismic features, such as thin shale barriers, can only be inferred and hence modeled through geologic concepts. These barriers may have a strong effect on fluid flow [4] and cannot be imaged by seismically.

To a large degree, effective parametrization of reservoir models is about the parametrization of geologic concepts. We will review different methods for parameterizing geologic concepts in Section 3 and propose new approaches in Section 4.

2.2 From Seismic to Simulation

Reservoir modeling typically starts with some understanding of the geology of the basin where a hydrocarbon reservoir resides. Depending on the stage at which the reservoir is being evaluated, different field data are collected. At the beginning of field development, data may include 3D seismic, well logs, and core analysis. As the reservoir enters into the production stage, more reservoir surveillance data become available, such as time-lapsed 3D seismic (commonly referred to as 4D seismic), well tests, and production history. The above data have very different resolution and spatial coverage, see Fig. 1. For instance, the areal and vertical resolutions of core data are ∼0.1 ft, and its range of coverage is ∼1 ft areally and hundreds of feet vertically. Note the wide spread of resolution and coverage of different types of data. As a comparison, the focus of today's research on reservoir performance prediction is based on reservoir models with resolutions of 10∼100 ft areally, and 1∼10 ft vertically. A central goal of reservoir modeling is to integrate these data into a common digital model of desired resolution.

First, faults and major stratigraphic units (bounded from top and bottom by horizons) are interpreted from the seismic data. The fault and horizon surfaces are then used to build a water-tight structural framework for the reservoir. This is a nontrivial task because the interpreted surfaces may not fit together, or they may be strongly distorted when the surfaces are transformed from the time domain to the depth domain (see Section 1). In practice, constructing the structural framework can take weeks or even months.

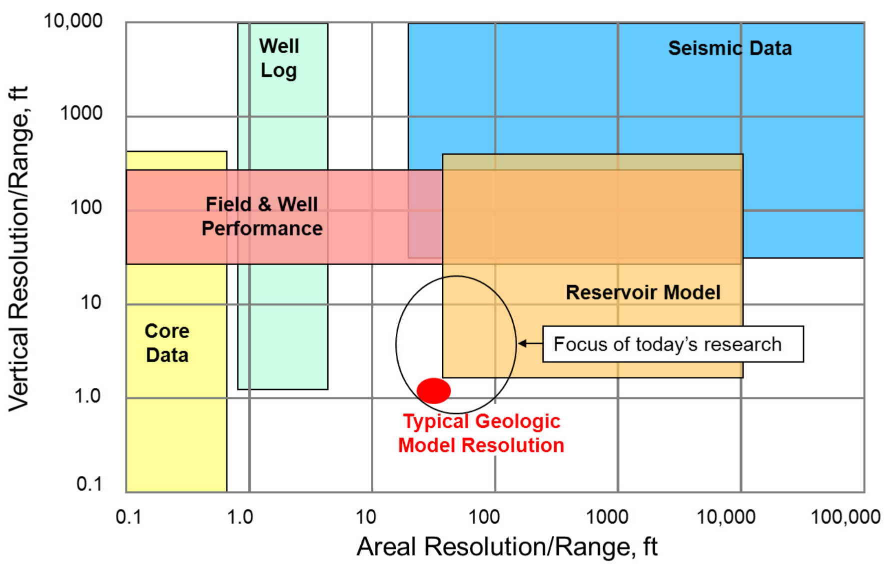


FIG. 1: Resolution and range of coverage of different data types used in reservoir modeling as indicated by the rectangular boxes: The left and bottom sides of each box indicate the horizontal and vertical resolutions, respectively; the right and top sides indicate the horizontal and vertical ranges, respectively.

The structural framework partitions a reservoir into volumetric compartments. These compartments determine the large-scale connectivity among different parts of the reservoir. Connectivity across faults is one of the major uncertainties in reservoir modeling. Whether a fault is sealing or not depends on many factors, such as the facies near the faults and the faulting processes. Sometimes, fault-seal analysis can be done to determine the range of possible transmissibility through faults. Uncertainty also exists in the location and geometry of the fault and horizon surfaces due to variations in the quality of seismic data. However, this type of uncertainty is seldom accounted for during uncertainty quantification because changing the frameworks is very time consuming.

When the seismic data can be correlated with well logs and core measurements through rock physics models, one may derive probabilistic spatial distributions of different facies, such as distributions of sand and shale in clastic reservoirs. The distribution may be three-dimensional when the seismic resolution is high and the reservoir is thick, or it may be two-dimensional because the seismic resolution is low and/or the reservoir is thin. For many deep-water clastic reservoirs, two-dimensional distributions of sand (or shale) thickness for major stratigraphic intervals are extracted. Seismic inversion on carbonate reservoirs are generally less successful because seismic waves travel much faster in carbonates, resulting in lower resolutions. In practice, the application of the seismically derived distributions (also known as seismic conditioning) is uneven because the correlation between seismic and well data is often uncertain and takes time to obtain. Another important product of interpretation is the geologic concept to be used for modeling subseismic features.

Interpretation is a labor-intensive activity, carried out manually in most cases. It involves complex pattern recognition based on the interpreter's understanding of the geologic setting as well as the petrophysical (and petrochemical) data gathered from wells. Interpretation is often the most time-consuming aspect of reservoir modeling, which in general takes months to accomplish depending on the amount and quality of data, the complexity of the reservoir, as well as what interpreters decide to interpret. Not knowing what geologic features might control the performance of a reservoir, interpreters tend to err on the side of describing more geologic details. However, for the same reason the interpreters may focus on the wrong set of details. Consequently, a very detailed interpretation may still lead to poor performance predictions because the details that characterize reservoir performance are missing [5].

Once the structural framework is constructed, the next phase is to model geologic concepts within the framework. The standard approach is to use geocellular grids and geostatistics. The modeling proceeds in a hierarchical manner, in the order of EODs, facies, and rock properties. The EOD boundaries are usually interpreted from seismic and/or well data. The degree of their uncertainty depends on the quality of seismic data and the number of well controls. Despite the uncertainty, EODs are typically modeled manually. Uncertainty in EOD boundaries has been difficult to include in the uncertainty analysis because there lacks an efficient way to change their geometry and cascade the change down through the hierarchy.

Facies (or RRTs) are modeled stochastically, sometimes with prescribed trends, and conditioned to seismic and well data by using geostatistical methods (see, e.g., [6, 7] and references cited therein). The most widely used methods are variogram-based methods that rely on two-point statistics (i.e., variogram). The variograms are not measurable because data are scarce; they are typically based on studies of outcrops and past experience with similar depositional environments. In addition to variograms, facies proportions are used to constrain the model. A common practice is to combine data from wells to derive facies statistics and then apply it to the entire reservoir. In other words, facies distribution is assumed to be stationary in the reservoir. This assumption is rather strong and unrealistic in many depositional environments. Sometimes seismically derived facies probability volumes are used to control facies distribution. See Section 3.1 for further discussions on stochastic modeling.

Once the facies model is in place, porosity, permeability, and other rock properties are modeled using geostatistical methods [7]. As shown in Fig. 1, core measurements are obtained using rock samples at a scale orders of magnitude smaller than the typical reservoir model resolution. Bridging this large-scale gap is challenging, especially for flow-related properties, such as permeability. Rigorous scale-up of these properties requires solving flow equations, which in turn requires models of fine-scale geology from very sparse data samples. Again, geologic concepts must be used to create these fine-scale models. Moreover, because the scale gap is so large, a multistage upscaling scheme may be required [8]. In practice, reservoir models at intermediate scale may be constructed to obtain statistics of rock properties at a larger scale [9].

We note that in Fig. 1 geologic models are distinguished from reservoir models. This reflects the common practice today; that is, one first constructs a geologic model on grids with millions of cells and then coarsens the model to form a reservoir model for simulation purposes. The main purpose of coarsening is to speed up the simulation runs. See [10–12] for reviews on various coarsening, upscaling, and multiscale methods developed over the past two decades for speeding up simulations. The fine-grid reservoir model is traditionally referred to as a geologic model. Traditionally, the geologic model is regarded as the "truth" or "reference" model. This status of geologic model is artificial, given the amount of uncertainty is involved. Therefore, we will not make the distinction here.

In addition to rock properties, reservoir fluid properties are also needed before the reservoir model can be simulated. Field measurement of fluid contacts and lab measurement of displacement functions, such as capillary pressure and relative permeability, are used to populate the reservoir model. The specific procedure will not be discussed here, although we point out that uncertainty also exists in fluid properties and may significantly impact reservoir performance. In the remainder of the paper, we will focus on modeling challenges related to geologic concepts and rock properties.

2.3 Calibration of Reservoir Models

To reduce uncertainty, reservoir models are calibrated (or history matched) against field and well performance data. The process is also known as history matching. First, uncertain model parameters are identified and selected. Then, the parameters are reduced based on their impact on reservoir performance. Finally, various optimization or Bayesian inversion techniques are used to adjust the parameters so that production history is matched. Note that both of the first two steps depend strongly on the parametrization of reservoir models.

The parameters involved in practical history matching are typically porosity and permeability multipliers in different reservoir regions and transmissibility multipliers across faults and stratigraphic boundaries. Sometimes, fluid properties are also adjusted. To be consistent with the geologic concepts, EODs or facies are often used to define the multiplier regions.

The multipliers are used when the underlying realization of the geologic concept (i.e., EODs and facies) remains unchanged during history matching. Often, it is important to update the conceptual realization while preserving the geologic concept. This can be difficult depending on how the geologic concepts are parameterized. For example, the EOD boundaries are typically modeled geometrically. Thus, changing it would require flexible geometric parametrizations, which usually is not available. Alternative parametrizations using geostatistics are possible. We will look at these options in more detail in the Section 3. Suffice it to say, modifying EODs is mostly done manually today and can be time consuming.

It should be pointed out that changing the structural framework of a reservoir model often triggers a complete rebuild of the model, which is even more time consuming. Therefore, the structural frameworks are kept the same in most model calibration cases. This is a serious constraint on reservoir modeling. We will not treat this topic further in this paper.

The calibrated reservoir models are used to predict future productions. In order to capture uncertainty, multiple calibrated models for different geologic scenarios are desired. However, in practice most of the effort is focused on obtaining one history-matched model because it still takes several months to get single match. With access to increasingly powerful parallel computers and automated workflows, the time required for history matching is shortening. Multiple history matches have been demonstrated on real fields (see, e.g., [13]).

2.4 Predictions and Uncertainty Quantification

As mentioned in the Introduction, the goal of reservoir modeling is to predict reservoir performance reliably. This is a shift from the historical view that reservoir modeling should give accurate predictions. To a large degree, the shift is due to the increased uncertainty in characterizing reservoirs in challenging geographic locations, such as deep-water and arctic environments. Because of high development cost, much fewer wells are used to produce from these reservoirs; reservoir modeling uncertainty increases significantly with decreased well control. Also, large capital investments are often decided early in field development when only a few wells have been drilled and uncertainty is high. For these reasons, it becomes increasingly important to quantify the uncertainty in reservoir performance.

Model-based uncertainty analysis faces the same challenge as those outlined above for model calibration [i.e., the lack of efficient parametrization of large-scale features (EODs/facies) of the reservoir model for automated sampling of geologic scenarios]. Moreover, compared to model calibration, uncertainty analysis requires more extensive sampling of the parameter space. In calibration, model parameters can be screened for their impact on specific performance measures; thus, sampling can be performed in a parameter space with reduced dimensions. In contrast, to quantify model uncertainty, one must also consider the parameters that have not shown an impact on performance through production history. These parameters may become important in the later stage of production or when production changes due to well management or additional in-fill wells. The dimension of the parameter space depends on the known complexity of the reservoir, such as number of faults, as well as the complexity of the more uncertain geologic concepts and their parametrizations. Models with >70 parameters is not uncommon. This number grows rapidly with the geologic features to be included in the analysis.

One important result of uncertainty analysis is the probability distribution of the reservoir performance over the parameter ranges. The wider the parameter range (due to uncertainty about the range) is, the more important it is to characterize the performance distribution in addition to the range. The uncertainty analysis should also produce an understanding of the geologic scenarios corresponding to the tail ends of the distribution, so that targeted data gathering can be performed to further evaluate the risks. Interestingly, the need to understand geologic scenarios is one of the reasons why methods based on stochastic PDEs, which are popular in the hydrology community, have not found wide applications in the petroleum industry. In any case, extensive sampling of the parameter space is likely required. As shown by [14], the widely applied experimental design methods in the industry are inadequate to capture the nonlinear model responses and, hence, the correct performance distribution.

There are many other forms of uncertainty and risk analysis that require less extensive sampling of the parameter space. For example, finding the range of reservoir performance can be formulated as a global optimization problem, whose solution requires targeted sampling in the parameter space. The amount of sampling may be less than that required by computing the probability distribution. Another example is to find geologic scenarios that would lead to certain fluid (e.g., water) production beyond the handling capability of a given facility design. The geologic scenarios can be used to plan risk mitigation strategies. Again, the sampling in a subset of the parameter space rather than the whole space is required. To the authors' knowledge, this kind of application of reservoir modeling is not common in the industry today. However, we believe they represent an important class of problems reservoir modeling should address.

Targeted sampling or not, given the large number of potential parameters in a reservoir model and the fact that many of them are highly uncertain, it is pivotal to judiciously choose the model parameters so that the dimension of the parameter space is as small as possible. How to achieve this goal is the focus of the rest of the paper.

3. PARAMETERIZATION METHODS IN RESERVOIR MODELING

According to [15], parametrization of a physical system is the "discovery of a minimal set of model parameters whose values completely characterize the system (from a given point of view)." In the context of reservoir modeling, ideally we would like to "select the least complicated model and the grossest reservoir description that will allow the desired estimation of reservoir performance" [16]. This is easier said than done.

For reservoir modeling, the reality has been and still is "the tendency toward state-of-the-art complexity" which often leads to the use of overparameterized reservoir models [17]. The parameters that do not have an effect on production are a major difficulty for history matching [18]. Because there lacks a priori understanding of what geologic features control fluid flow in a given production scenario, the apparent logical response is to model reservoirs as detailed as possible so that nothing important would be missed. But this is costly, both in terms of time and effort. More importantly, contrary to the general belief, putting more details in reservoir models is not more likely to produce better predictions unless the details that matter to flow are modeled.

In recent years, progress has been made toward recognizing geologic features that are important to fluid flow. An interpretation paradigm shift from traditional seismic attribute extractions and geostatistical methods focusing dominantly on net sand distribution to conceptual approaches that better capture the spatial organization of net reservoir facies and internal flow barriers has been suggested [5]. As an example, Fig. 2 shows the distribution of different EODs in a cross section through a deep-water channelized model. This distribution is based on the conceptual understanding of deep-water channelized systems, where, for example, the axis EOD has high porosity and permeability and margin EOD has lower porosity and permeability. It also has been observed that seismically conditioned, geostatistical models of deep-water reservoirs were unable to match reservoir behavior observed in 4D seismic or production data. A parallel study in [19] also showed the importance of modeling the organization and architecture of reservoir facies in additional to their proportions. On the other hand, the significance of one geologic feature may vary depending on the measure of subsurface flow. For example, in appraisal and development studies where gross field production is the main concern, reservoir architecture may not be an important uncertainty unless reservoir connectivity or tortuosity are affected [20].

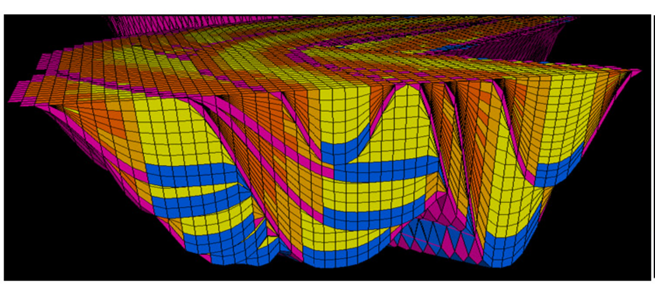
Simply put, the pertinent reservoir geology and, hence, the minimal set of geologic parameters that completely characterize a reservoir vary with the depletion scenario and the objective of reservoir study. The key questions to be addressed are (i) how to effectively parameterize static geologic concepts, (ii) how to reduce the parametrization with respect to given flow dynamics, and (iii) how to extract generic learnings from (ii) so that the next modeling effort can start at a higher ground? In the following, we present a brief overview of existing parametrization methods and discuss their advantages and limitations with respect to the three questions.

3.1 Parametrization Methods: A Brief Overview

Variogram-based geostatistical methods are the workhorse in the petroleum industry to parameterize geologic concepts, especially facies distributions (see, e.g., [6, 7]). These methods can produce fairly complex facies distributions. However, they assume stationarity and have difficulty modeling realistic geologic concepts with complex, curvilinear, and continuous structures. For this reason, methods based on multipoint statistics (MPS) were developed [21, 22]. These methods can better handle complex patterns; however, handling nonstationarity remains a challenge. See [23, 24] for recent development in this area.

The advantage of statistical methods is their ability to condition to hard data (e.g., wells) as well as secondary information, such as seismically derived facies probabilities. Their "pixel-based" (or voxel-based to be more precise) approach provides much flexibility in representing complex facies patterns. However, a major drawback of statistical methods is that enforcing facies continuity is difficult, especially for long, thin facies such as channels [25] and minority facies such as thin shale barriers draping on the channel boundaries as shown in Fig. 2. Another issue with statistical methods is the lack of continuous parametrization that is very useful in model calibrations. This latter difficulty has been addressed by [26, 27] and [28], and more recently by various parameter-reduction methods (see Section 3.2).

For continuous features, geometric parametrization is natural and very efficient in terms of number of parameters required. Geologic shape parametrization based on flexible geometric representations have been proposed in [29] and are recently applied to fluvial environments using event-based modeling that mimic the depositional process [30]. See



	EOD	Porosity	Permeability (md)
	Margin	0.1	10
	Off-axis	0.2	100
	Axis	0.3	1000
	Axis – gravel	0.3	10000
	Shale	0	0

FIG. 2: Organization and averaged property of different EODs in a deep-water channelized environment. Note that the thin shale drapes on the boundaries of the channels have significant impact on fluid flow.

also [31] for a review on object- and process-based modeling methods. Conditioning object-based or process-based models to hard data is difficult, especially when the number of constraints is large. Also, the stochastic simulation of objects render the parametrization discontinuous. MPS methods represent an attempt to resolve this problem by using geometrically generated models as training images for collecting statistics at various scales. In doing so, the advantage of the compact geometric parametrization is lost in the "rasterization" of the geometries of reservoir heterogeneity.

The recognition that many important geologic features are often associated with depositional and erosional surfaces [5] has led to the development of stochastic surface modeling approaches [32–34]. These methods use explicit surfaces to delineate depositional or erosional boundaries. The surfaces thus provide the subspace on which those thin shale barriers in Fig. 2 can be modeled. Moreover, the surfaces define volumes for further pixel-based statistical modeling. Along this line, level-set methods with stochastic velocity models are used to model or calibrate channel and facies boundaries [35–37]. Other hybrid geometric and stochastic pixel-based approaches are developed to maintain facies continuity [25, 38]. It remains to be seen if these hybrid methods will overcome the difficulties faced by geometric and pixel-based methods.

3.2 Parameter Reduction through Static Compression

Various parameter reduction techniques have been developed for pixel- or voxel-based parametrization of reservoir models. Most of these developments focus on continuous variables such as porosity and permeability, instead of EODs and facies. Similar to image compression, this class of methods is based on compressed representation of rock property distributions on a geocellular grid. Let $\mathbf{x}_i \in R^m$ ($i = 1, \ldots, n$) be a set of centered (i.e., zero mean) realizations of a random field. We seek $p \ll m$ basis functions such that $\mathbf{x}_i$ can be approximately represented as the linear superposition of the basis functions

$$\mathbf{x}_i \approx \mathbf{A}\mathbf{y}_i \qquad (i = 1, \ldots, n),$$

where $\mathbf{A}$ is an $m \times p$ matrix whose columns are the basis vectors and $\mathbf{y}_i \in R^p$ is the expansion coefficient. Let $\mathbf{X}$ and $\mathbf{Y}$ be the matrices with column vectors $\mathbf{x}_i$ and $\mathbf{y}_i (i = 1, \ldots, n)$, respectively. In matrix form, we have

$$\mathbf{X}_{m\times n} \approx \mathbf{A}_{m\times p}\mathbf{Y}_{p\times n} \qquad (p \ll m).$$

In other words, the problem is to find a low-rank approximation to $\mathbf{X}$.

Singular-value decomposition (SVD) is a popular choice for this task. Let the rank of $\mathbf{X}$ be k. Thus, $k \leq \min(m, n)$ and

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T,$$

where $\mathbf{U}$ is an $m \times k$ orthonormal matrix, $\mathbf{V}$ is an $n \times k$ orthonormal matrix, and $\mathbf{\Sigma} = \text{diag}(\sigma_1, \ldots, \sigma_k)$ is a $k \times k$ diagonal matrix. The numbers σ_j are the singular values of $\mathbf{X}$ and are arranged in weakly decreasing order

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_k \geq 0.$$

Thus, a rank-p ($p < k$) approximation can be obtained by setting $\sigma_j = 0$ for $j > p$. That is

$$\mathbf{A}_{m\times p} = \mathbf{U}_{m\times p}\mathbf{\Sigma}_{p\times p} \quad \text{and} \quad \mathbf{Y}_{p\times n} = \mathbf{V}^T_{n\times p}. \tag{1}$$

Because $\mathbf{V}_{n\times p}$ is orthonormal, we have

$$\sum_{j=1}^{n} \mathbf{y}_j\mathbf{y}_j^T = \mathbf{Y}_{p\times n}\mathbf{Y}^T_{p\times n} = \mathbf{V}^T_{n\times p}\mathbf{V}_{n\times p} = \mathbf{I}_{p\times p}. \tag{2}$$

Note that $\mathbf{y}_i$ can be considered as a realization of a random vector in R^p. Then, Eq. (2) means that the components of the random vector are uncorrelated. Recently, efficient randomized algorithms have been developed for truncated SVD that can handle large data sets. See [39] for an excellent review.

The above construction of $\mathbf{A}$ in (1) is equivalent to the principal component analysis (PCA) in statistical learning or a finite dimensional Karhunen-Loève expansion (KLE). Indeed, $\mathbf{U}$ are eigenvectors of $\mathbf{X}\mathbf{X}^T$, which is proportional to the empirical covariance matrix for the random realizations $\mathbf{x}_i$. Thus, the KLE can be used to generate realizations with the same covariance as that of $\mathbf{x}_i$ using a set of uncorrelated random variables. In practice, the number of realizations n is typically smaller than m. Using KLE to derive stochastic PDEs for subsurface flows for log-normal permeability fields is well established. Applying KLE as a way to parameterize the reservoir model in history matching appears to begin with [40] (see also [41]).

Other types of basis functions have also been used to form $\mathbf{A}$. For example, using multiresolution analysis based on Haar wavelets is proposed as a way to reparameterize the permeability field and wavelet coefficients that are sensitive to the history data are identified and used for history matching [42–45]. Similarly, discrete cosine transform (DCT) is used to construct $\mathbf{A}$ in [46], where the advantage of DCT over KLE in representing non-Gaussian channelized features was demonstrated. DCT basis functions are independent of model realizations, although they can be trained with respect to a given set of realizations [46]. The main advantage of DCT is the availability of fast transform algorithms. Application of DCT to history-matching problems is shown in [47, 48]. In [48], sparsity constraints on the expansion coefficients is enforced through l^1 minimization. Note that DCT only applies to tensor product grids and hence has limited practical applications. However, generating basis vectors on unstructured grids is recently developed [49].

Compressing $\mathbf{X}$ using linear combinations of basis functions is somewhat limited. In [50], nonlinear PCA is used to parameterize $\mathbf{X}$ in a feature space. The feature space is created by using a nonlinear mapping $\boldsymbol{\phi}(\mathbf{x})$, with $\mathbf{x} \in R^m$, $\boldsymbol{\phi} \in R^{m_f}$ and m_f being typically much larger than m. Various nonlinear mapping can be used to generate the feature space. Polynomials are used in [50] because they correspond to higher-order multipoint statistics. Then, PCA is applied on samples $\boldsymbol{\phi}(\mathbf{x}_i)$ in the feature space. Despite the large dimension of the feature space, PCA can be done efficiently by using the celebrated kernel trick [51] so that the feature space needs not be formed explicitly. Applications of this approach to history matching have been demonstrated in [52, 53].

3.3 Remarks on Compression-Based Methods

Although the image compression method represents a general approach toward parameter reduction, it does have some shortcomings. For practical applications of these methods, one challenge is to relate the basis functions (i.e., eigenvectors of some discrete operator) to the geologic concepts underlying the model realizations. The parametrization through coefficients of basis functions is inherently nongeological. Consequently, one may successfully conduct history matching using the compressed representation without knowing what geologic changes have been made to achieve the match.

Another limitation of these methods is that some important fine-scale features, such as continuous thin shale barriers (Fig. 2), can be difficult to capture using the basis functions. These features are often represented as properties on certain conceptual surfaces in practice. Even if they are represented explicitly on grid cells, the challenge will be to construct basis functions to preserve the continuity of the thin barriers, a critical factor determining the effect of the barriers on fluid flows.

Regardless of the basis functions used, the compression takes advantage of the fact that the reservoir models have large continuous features. The compression essentially filters out features of short correlation lengths [50]. Typically, the large-scale features have a dominant effect on flow. On the other hand, static compression will not work if the reservoir model only contains features of short correlation lengths: the singular values decay slowly and all singular vectors must be used. Yet, it is well known in reservoir engineering that such models are "so heterogeneous that they behave homogeneously." In other words, model parameters can be greatly reduced by using a few effective parameters, such as effective permeability. Therefore, compression by itself does not address the main parametrization challenge, namely, representing effective geologic features with respective to flow.

It remains to be seen if the compression-based methods can maintain the spatial organization of facies distributions, which can be important to fluid flow [5, 19]. Thus far, the conceptual models used to demonstrate these methods are rather simplified. More validation is require by applying these methods to more realistic geologic concepts.

3.4 Distance-Based Modeling and Dynamic Compression

Another way to measure model similarity is to use a distance (or metric) defined between models [54, 55]. In particular, the distance may be based on the difference between the production profiles of two model realizations [56]. Thus, the similarity is based on aggregated dynamic information (e.g., at the wells). As a result, two models having a small distance between them may not look alike from the static point of view. In this approach, a distance matrix $\mathbf{D}$ is formed from $d(\mathbf{x}_i, \mathbf{x}_j)$ $(i, j \in [1, n])$, where d is a distance function that may require flow simulations. The models are then mapped into a low-dimensional metric space through multidimensional scaling (MDS) so that the Euclidean distance in the metric space is a good approximation of the model distances in the physical space. Kernel PCA is then used to transform the models to yet another metric space with further dimension reduction. If differences in production profiles are used to define the distance, then the truth model can be characterized in the metric space by using its distances to other models. This fact has been used to solve history-matching problems. See [57] for a more detailed review of the process.

The ability to use flow measures as distances is an attractive feature of distance-based modeling. However, the methodology does not seek parametrization of the static reservoir models. Instead, parametrization of the relative positions of models is done in the abstract metric space. In the following, we will focus on parametrizations of the static reservoir models but using flow information.

4. HEURISTIC MULTISCALE PARAMETERIZATION

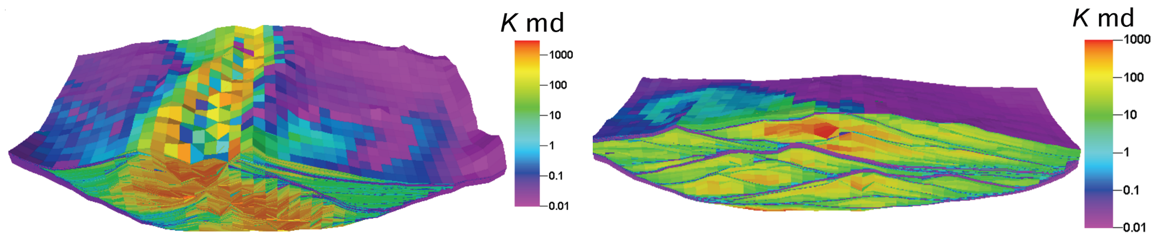
If the gist of image compression is to extract the continuous features from realizations of geologic concepts and if the impact of features with short correlation lengths on flow can be captured by coarse-scale effective properties, then it is natural to ask if parametrization of geologic concepts can be done more directly by taking both facts into account. In this section, we test the feasibility of combining this understanding with geologic hierarchies to form a heuristic parametrization method. Because of the geologic hierarchy, the resulting parametrization is inherently multiscale and closely follow the geologic concepts. We are interested in developing an approach to reduce model parameters based on flow simulations.

In the following, we show that combining surface representation of thin barriers and volumetric trend functions may give rise to effective parametrization. We also show that completely filtering out features of short correlations may not be always desirable. To achieve parameter reduction and maintain flow behavior, multiscale parametrization is needed. We show that this can be done locally near the wells to preserve the characteristics of well-driven flows. Our parametrization is similar to surface-based modeling methods (e.g., [32–34]) in the use of volumetric trend functions. However, a significant difference is that we use trend functions as a way to reduce model parameters. This is reflected in our use of permeability trend functions instead of stochastically generated permeability field as in, e.g., [32].

4.1 Parametrization Based on Geologic Hierarchy

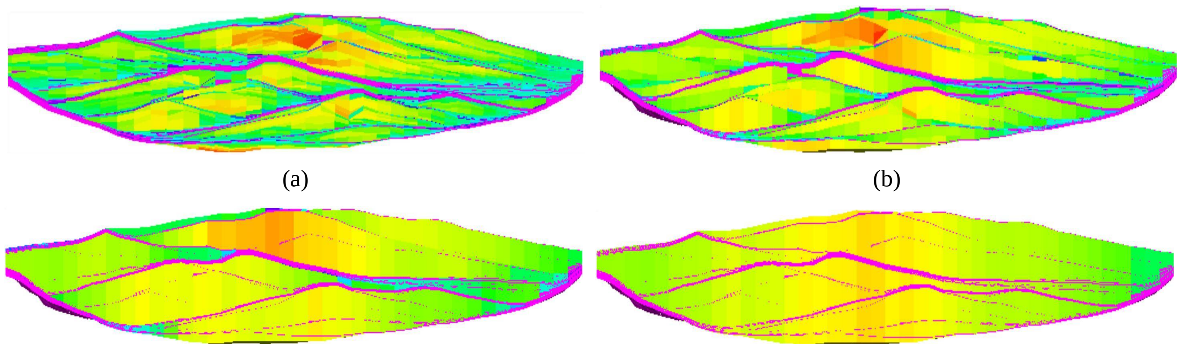
Many clastic reservoirs exhibit hierarchical structures. In this section, we study the hierarchical structure of the geobodies, its control on reservoir heterogeneity, and the impact of different hierarchies on reservoir performance. Because it is difficult to get a reservoir model with realistic heterogeneity on all scales, we employ a process-based numerical model [2]. The model is formed by simulating the sedimentary process during flooding from a river mouth (orifice) into a basin. Figure 3 shows medial (close to the orifice) and distal (further into the basin) sections of the model.

Process-based models are based on solving the PDEs that govern the process of deposition and erosion that generate detailed distribution of grain size in a depositional system (see [2] for the details on the governing PDEs). The grain size distribution is then used to estimate the porosity and permeability in the field. This physical process creates depositional bodies at different hierarchy as the deposition location changes. We analyze our process-based reference model (Fig. 3) by successively removing the heterogeneities in the lowest hierarchy as described below. Figure 4(a) shows a cross section of the reference model in the distal section. Here, we can see that there are depositional bodies of different scales. The smallest of those depositional bodies called storey are formed by one single flow event. Small changes in the location of the feeder channel (caused by small avulsion) generate a sequence of lithologically similar



(a) (b)

FIG. 3: Process-based model used as reference in this study: (a) Medial section and (b) distal section. Figures are vertically exaggerated.



(a) (b)

(c) (d)

FIG. 4: (a) Storey level sand hierarchy model with all the detail as process-based model, (b) upscale model with 54 storey-sets, (c) upscale model with five complexes, and (d) upscale sand model at a complex-set detail. In all the models, shales are at storey-set level. Property shown is log(k) in mD with a range between 0.01 to 1000 and figures are vertically exaggerated.

sedimentary bodies stacked on each other forming a storey set, also called a lobe, as shown in Fig. 4(b). Geobodies within a storey set thicken, thin, or aggrade vertically. Bigger avulsions results in lobe complexes or simply called complexes [Fig. 4(c)], which are characterized by alternative thick sandstone, thin sandstone, and mudstone packages. Even bigger avulsions result in lobe complex sets or simply called complex sets [Fig. 4(d)].

At each hierarchy, the porosity and permeability is generated by upscaling the lower hierarchy features of the reference model. In Figs. 4(b)–(d), the upscaling is done by simply averaging the properties along the vertical direction. Although this upscaling method is just simple averaging, the areal property trends are preserved at their respective hierarchies. When upscaling sand permeability from the reference model to, for example, the complex level, a single permeability value is obtained for an areal position in a complex. In reality, there are vertical permeability trends within a complex and they can be modeled using very simple trend functions. Example vertical trends at the complex level can be channels in complexes are bottom loaded and depositional parts of complexes are top loaded. Figure 5(b) shows a simple triangle-shaped function to model such variations in the vertical direction. When the trend in Fig. 5(b) is applied to each complex in Fig. 4(c), we obtain Fig. 5(a). Similar trends can be applied to other hierarchy of geobodies. It will be shown that this simple vertical trend can be used to fine-tune the reservoir performance at a given hierarchy.

The shale content in the different hierarchies is shown in Fig. 6, which is a good way to look at the different level of details that can be included in the models. We can clearly see that larger hierarchy shales are laterally more extensive and thicker and are expected to have more control to the performance of the reservoir.

We carry out reservoir simulation on these different models to determine which level of detail is needed to predict the reservoir performance. Because the shale in the medial section is less continuous due to larger erosional effects, the simulation is carried out separately for medial and distal sections. For each section, six models of different levels

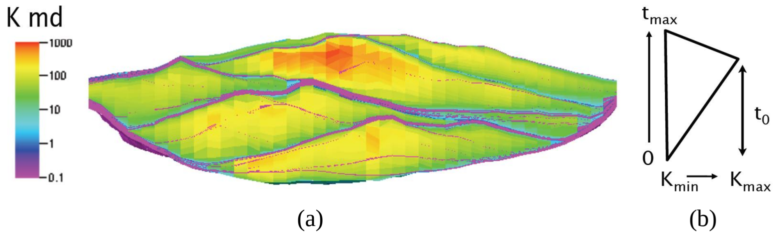


FIG. 5: Complex model with vertical trend: (a) Permeability of complex model with vertical trend and (b) triangular trend function used to generate (a).

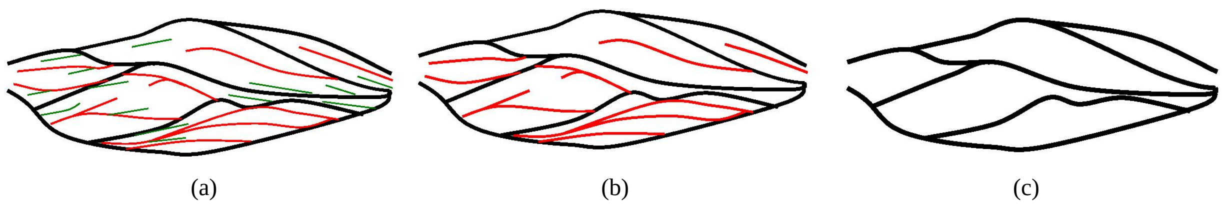


FIG. 6: Digitized shale in the distal updip section at three different level of hierarchy. (a) Storey level, (b) storey-set level, and (c) complex level. Larger hierarchy shales are laterally more extensive and thicker. Storey level shales incorporate all the shales.

of details are compared: storey model (base case), storey set, complex with vertical trends, complex, complex set, and a tank model with all internal heterogeneity averaged out. Different levels of shales (Fig. 6) can also be included in the models. In this study, three injectors were used for both medial and distal sections along with five to six producers. Gas and water are injected as two possible scenarios to understand the effects of mobility. All wells were active from the start of production and operated on bottom hole pressure constraints. The production wells would be shut in when the production rate fell below a certain value. The simulations were run for a maximum of 45 years. See [58] for more detailed description of the simulation study. Here, we focus on results of particular interest to parametrization.

The two images in Fig. 7 show the recovery curves for water and gas flooding, respectively, in the distal section. For both the fluid types, a very similar recovery pattern versus level of hierarchy is produced. As the amount of detail increases, the recovery decreases and the complex level model with vertical trends is very close to the detailed, base-case model. Complex and complex with vertical trend models have shales at storey-set level (i.e., finer scale shales are not modeled), whereas other models have all levels of shales. Of course tank model does not have any shale. This

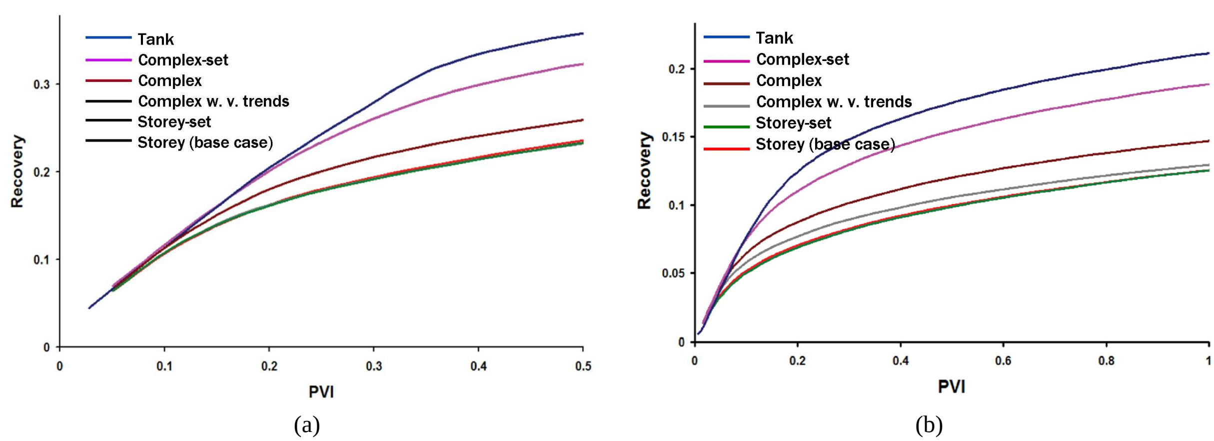


FIG. 7: Recovery curves in the distal section: (a) Water drive and (b) gas drive.

signifies that storey-level shales need not be incorporated in the geomodels and storey-set-level shales with complex level sands (with vertical trends) are enough to capture the detailed model flow responses. This is consistent with the medial section results, except that complex level shales are important in the medial section, and storey-set-level shales are important in the distal section; for the distal section, including only complex level shales, proved to be insufficient (results not shown). This difference is because those levels of shales are continuous in those respective sections relative to the well spacing.

As mentioned above, the results for the medial section is consistent with distal section. Thus, we will not show the same comparison for the medial section. Because the results indicate that complex-level model with vertical trends is a sufficient approximation to the reference model, we would like to test the sensitivity of the reservoir performance to the vertical trend function. Figure 8 shows the recovery curves of medial section under gas flooding. The vertical trend function is adjusted to shift the location of the maximum value while preserving the triangular shape. It is interesting to find out that by simply adjusting the trend function, the response of the reservoir is able to approach the reference solution from both above and below. This can be used as an easy measure to assess the uncertainty of the reservoir model after proper level of heterogeneity is parameterized.

4.2 Localized Fine-Scale Parametrization

Heterogeneities are present in a wide range of scales in petroleum reservoirs. Larger scale heterogeneities can be more easily parameterized based on geologic concepts and measurement, such as EODs and facies boundaries. On the other hand, smaller scales are associated with more significant uncertainties and their parametrization is more difficult. The only exception in petroleum reservoirs are the regions near the wells, where direct measurements (e.g., well logs and core analysis), are possible. In this section, we will show that localized fine-scale parametrization at near well regions can have a significant effect on the modeling and prediction of reservoir performances.

The reservoir model used in this study is representative of a carbonate platform interior environment of deposition as shown in Fig. 9. All levels of heterogeneities are represented in Fig. 9(a), which is assumed to be the truth and considered to be the base case in our comparative study. However, in reality the small-scale heterogeneities are

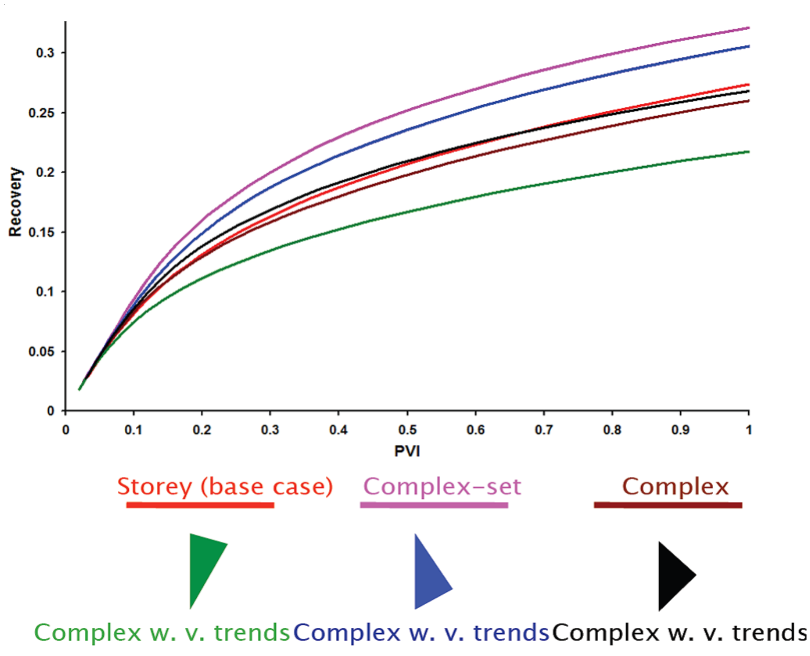


FIG. 8: Recovery curves in the medial section gas drive.

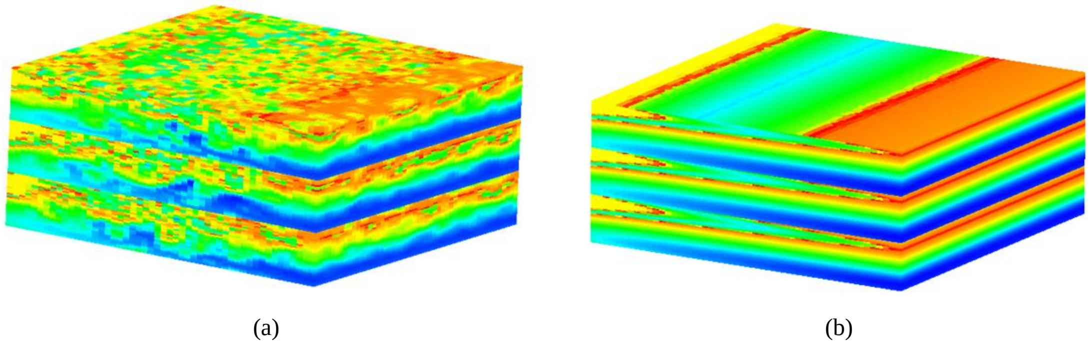


FIG. 9: Permeability distributions of a carbonate platform interior model with different levels of heterogeneity: (a) Base model where all levels of heterogeneities are represented and (b) comparison model where only large-scale heterogeneities are parameterized by a smooth trend function.

difficult, if possible, to measure or parameterize. They are usually modeled through a stochastic simulation process. Modeling and parametrization of large-scale heterogeneities can be relatively easily achieved by constructing certain smooth trend functions based on the understanding of the relevant geologic concept. Figure 9(b) shows a realization of such a model, whose properties approximate the average properties of the base model in Fig. 9(a).

In order to compare the predicting capabilities of the two models in Fig. 9, 49 producers and 36 injectors are placed in the reservoir using a typical five-spot pattern, as show in Fig. 10. The size of the reservoir model is $3000\times3000\times30$ m. Under the five-spot pattern, the resulting well spacing is about 45 acres. The 49 producers are grouped into seven well groups (designated as Platform A–G) to study the localized production characteristics. Platforms A and G are marked using ellipses in the figure. The rest of them are defined in the same manner from left to right in that order. The injectors are constrained by a constant injection rate, while the producers are constrained by a constant bottom hole pressure. Black oil simulation is performed until three pore volumes (PV) of water has been injected. Figure 11 shows the production curves of the two models. Solid lines are from the base model, and dashed lines are from the trend function model. We can see from Fig. 11(a) that the two sets of curves for whole field production are almost indistinguishable. Figures 11(b) and (c) show the production from two individual platoforms, B and D. Clearly, on the platform level, the productions from the two models do not match. Productions from other

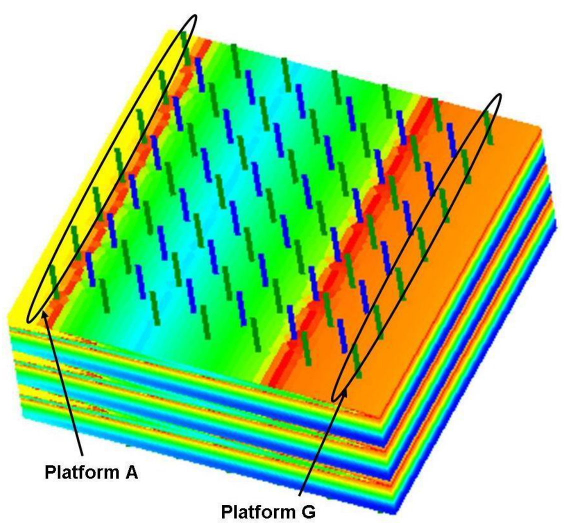
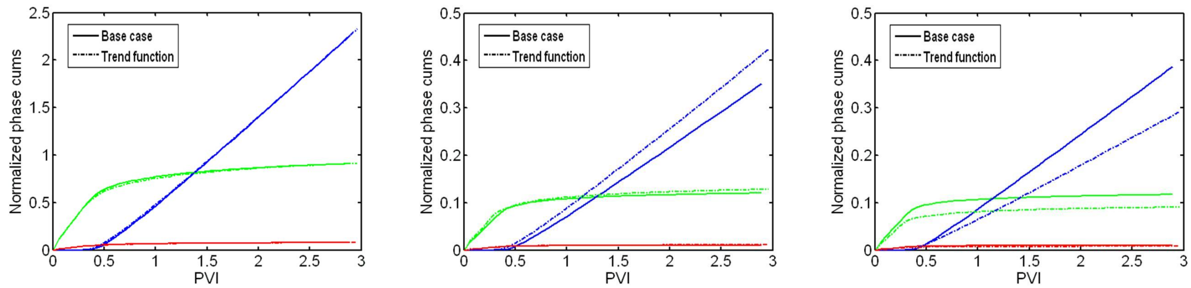


FIG. 10: Five-spot injection pattern used in the reservoir. Green wells are producers, and blue wells are injectors.



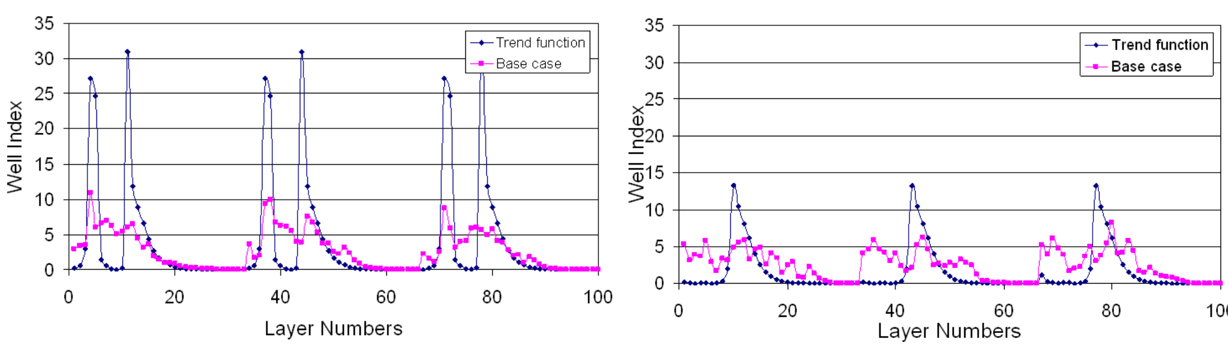
(a) (b) (c)

FIG. 11: Comparison of production of whole field and individual platforms. By convention, green, red, and blue curves represents oil, gas, and water productions, respectively. Also note that the axes are normalized by the reservoir pore volume. (a) Whole field production, (b) production from Platform B, and (c) production from Platform D.

platforms exhibit similar differences, and their production curves are omitted in this paper. This indicates that, for this model, parameterizing only the larger scale heterogeneities are sufficient to the prediction of whole field production behaviors, where the effects of small scale heterogeneities tend to average out and do not make remarkable difference as a whole. On the other hand, the mismatch at individual platforms show the need of additional localized details to better characterize the model and to improve the predictive capabilities of the model based on trend functions.

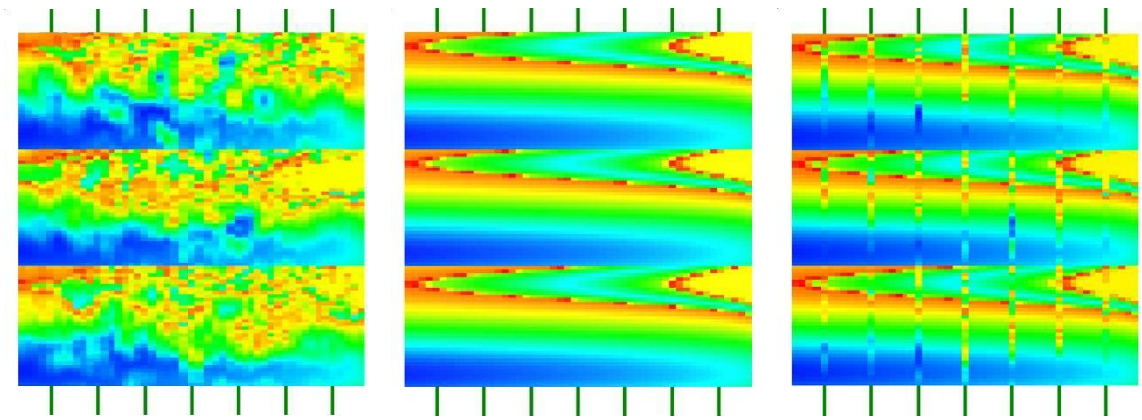
The model based on trend function produces more at Platform B, and less at Platform D. The variation of the well index with layers is plotted for the wells in Platforms B and D along the well tracks in Fig. 12. Well index is essentially the transmissibility between the well bore and the reservoir cells it connects with. It is determined from the well bore radius, and the shape and property of reservoir cells that are connected to the well. Clearly, there is a correlation between well index and the productivity of the wells comparing Figs. 11 and 12. For example, for the wells in Platform B, the well index from the trend function is generally bigger than those from the base case, which correlates with the fact that trend function model produces more than the base case at the platform. The difference for Platform D in Fig. 12(b) is less clear, but a simple calculation can show that the average well index along the well track of the base case is higher than that of the trend function, and thus correlates with the production curves shown in Fig. 11(c). Because the reservoir cells that are connected to wells have such significant impact on the productivity of the wells, the near well regions have to be characterized more carefully. Fortunately, this information is usually available in practice through various technologies, such as core sample analysis and well logging.

On the basis of the above discussion, we construct a new model by replacing the properties of the well-penetrated cells in the trend function model with respective values from the base model. Figure 13 show slices through the



(a) (b)

FIG. 12: Variations of average well index with layers for wells in Platform B and D: (a) Platform B and (b) platform D.



(a) (b) (c)

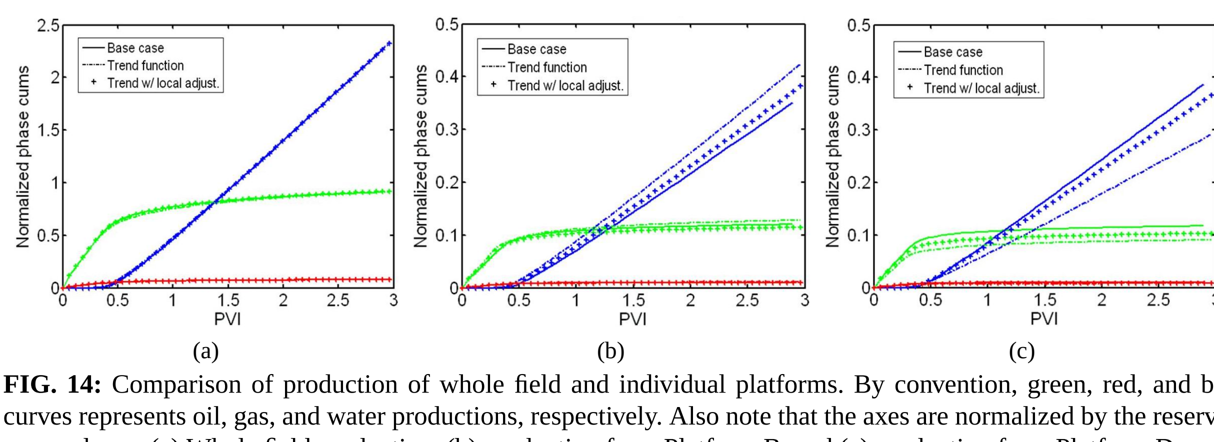
FIG. 13: Slices of the models through Platform D: (a) Base model, (b) trend function, and (c) trend function with local adjustments at wells. For the cells that are penetrated by wells, their properties come from the base model (a). The slices are vertically exaggerated.

location of the production Platform D, which is at the center of the models.We put this new model into production using the same facilities settings and compare the results to both the base case and the unmodified trend function in Fig. 14. The production results of the new model are represented using a plus sign; solid and dashed lines are from the base model and trend function, respectively. Same as before, the whole field productions are almost identical as shown in Fig. 14(a). At the individual platform level, however, the production results are significantly improved as compared to the original unmodified trend function.

This experiment shows that localized fine-scale parametrization can significantly improve the prediction capabilities of reservoir models, provided that the large-scale (global) heterogeneities are properly parameterized. Note that in this experiment the modification of properties (i.e., the localized fine-scale parametrization) is only applied to the cells penetrated by wells. Further improvements can be expected if the range of modification is extended.

5. CONCLUDING REMARKS

As in any other discipline, the fundamental challenge to reservoir model calibration and uncertainty quantification is the "curse of dimensionality." Reducing reservoir model parameters goes a long way toward reliable reservoir



(a) (b) (c)

FIG. 14: Comparison of production of whole field and individual platforms. By convention, green, red, and blue curves represents oil, gas, and water productions, respectively. Also note that the axes are normalized by the reservoir pore volume: (a) Whole field production, (b) production from Platform B, and (c) production from Platform D.

performance prediction. To this end, it is important to look at reservoir modeling problems from the perspectives of the practical questions the models are expected to address. Complicated geologic concepts may be simplified because many fine-scale details do not have a strong effect on flow or can be adequately represented by using simple effective descriptions at larger scales.

Although flow-based screening of model parameters with respect to specific flow scenarios and reservoir performance measures is important, the pressing question for the industry is how to generalize the learnings so that lessons need not be learned repeatedly. As [31] put it, “research that examines which scales of geologic structure affect flow and transport behavior would be valuable to guide the level of detail required in maps of hydraulic (or reservoir, added by the authors) properties.” The purpose of such a research is to build up the empirical guidelines instead of rigorous determinations. In only very special cases, can one prove the irrelevance of some fine-scale geologic features mathematically. In general, the impact of geologic features will have to be examined by experimentation using different flow scenarios, either numerically or otherwise. However, no amount of empirical evidence can prove that a geologic feature is always unimportant. Nonetheless, the empirical evidences may be used to build probabilistic models, such as Bayesian networks [59], to provide educated guesses.

As in any scientific endeavor, we must treat reservoir modeling as an iterative process with hypothesis, testing, and calibration. In this process, one first postulates what geologic details matter to flow and then tests the hypothesis by adding or removing details and examines the impact of these details on flow and transport. We favor a top-down approach for several reasons. First, empirically we know that the large-scale geologic features often have strong effect on reservoir performance. Thus, beginning with the large-scale, gross reservoir description is more efficient in terms of utilizing our empirical learnings. Secondly, the reservoir geology is less uncertain at the larger-scales and there is no ambiguity regarding the starting hypothetical model. Besides, the modeling process naturally proceeds from large to small scales. Thirdly, starting with the large-scale features enable parallel effort of interpretation and modeling. That said, in practice we can begin with any reasonable guess of the level of details, planning to err on the side of fewer details and parameters.

Whether further details are needed can be tested against specific questions of interest. These questions depend on the stage of field development and depletion. For example, in early development, we are interested in whether the distribution of reservoir performance will be changed significantly by adding further details. For example, we may ask if further details broaden the performance distribution and hence change our risk assessment. When production data become available, whether further details can help achieving better history matches will be the focus. As production goes on, more geologic details will likely be needed to provide nuanced explanation of production data. We note that the testing does not require direct comparison of two different parametrizations on a model-by-model basis. As in distance-based modeling [57], the comparison of models can be made in terms of their performance predictions.

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